2103000206020036
EXAMINATION FEBRUARY-MARCH 2024
BACHELOR OF SCIENCE (SIXTH SEMESTER)
MATHEMATICS-XI (MTH-606-NUMBER THEORY-II) -LEVEL 2

[**Time: As Per Schedule]** **[Max. Marks: 50]**

Instructions:
1. Fill up strictly the following details on your answer book
a. Name of the Examination : **BACHELOR OF SCIENCE (SIXTH SEMESTER)**
b. Name of the Subject : **MATHEMATICS-XI (MTH-606-NUMBER THEORY-II) – LEVEL 2**
c. Subject Code No : **2103000206020036**
2. Sketch neat and labelled diagram wherever necessary.
3. Figures to the right indicate full marks of the question.
4. All questions are compulsory.

Seat No:

Student's Signature

Q.1 Answer Five any of the following: **10**

1. Write the condition that the linear congruence ax≡b(mod n) has a solution

2. Converse of Fermat's little theorem is true? Justify your answer?

3. Find the value of $\sum_{d|30} d$ and $\sum_{d|30} 1$

4. If n is an even integer then prove that ϕ(2n)=2 ϕ (n)

5. Solve $36x \equiv 8(\mathrm{mod}\ 102)$

6. Find the reminder when 15! is divided by 17

7. Find the all forms of positive integers satisfying τ(n)=10, what is the smallest positive integer for which this is true

8. If n=$2^a 3^b; a, b \in Z$ then prove that $\phi(n)|n$

Q.2 Answer any two questions: **10**

1. Solve the linear congruence $34x \equiv 60 \pmod{98}$

2. State and prove Chinese Reminder Theorem

3. Find two consecutive integers such that first has square factor and second has cube factor.

Q.3 Answer any two questions: **10**

1. If p is prime then prove that $a^p \equiv a \pmod{p}$ for any integer a.

2. (i) For prime p show that $1^{p-1} + 2^{p-1} + \cdots + (p-1)^{(p-1)} \equiv -1(mod\ p)$.

   (ii) For odd prime p show that $1^p + 2^p + \cdots + (p-1)^p \equiv 0(mod\ p)$

3. Define Absolute pseudo prime. Prove that 561 is an absolute pseudo prime.

Q.4 Answer any two questions: **10**

1. If $2^k - 3$ is prime then show that $n=2^{k-1}(2^k - 3)$ satisfies $\sigma(n)=2n+2$

2. For any positive integer n prove that $\sum_{d|n} u(d) = \begin{cases} 1; if\ n = 1 \\ 0; if\ n > 1 \end{cases}$

3. Prove that $\left[\frac{n}{2}\right] - \left[-\frac{n}{2}\right] = n$; for all integer n.

Q.5 Answer any two questions: **10**

1. Let n is a positive integer and gcd(a,n)=1 then $a^{\phi(n)} = 1(mod\ n)$ and hence deduce the Fermat's theorem

2. For n>2, prove that $\phi(n)$ is an even integer

3. Verify that $\varphi(3n)=3\varphi(n)$ iff $3|n$

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